Exercise 5

Show that the function $f(x) = -2(x-1)^2 + 3$ is not one-to-one.

Solution

To show that the function is not one-to-one, try to find its inverse. Switch x with y in the given formula.

$$x = -2(y-1)^2 + 3$$

Solve for y.

$$x - 3 = -2(y-1)^2$$

$$\frac{x-3}{-2} = (y-1)^2$$

$$\sqrt{\frac{x-3}{-2}} = \sqrt{(y-1)^2}$$

$$\sqrt{\frac{x-3}{-2}} = |y-1|$$

Remove the absolute value sign by placing $\pm$ on the left side.

$$\pm\sqrt{\frac{x-3}{-2}} = y - 1$$

Add 1 to both sides.

$$\pm\sqrt{\frac{x-3}{-2}} + 1 = y$$

There are two possible formulas for the inverse function, $\sqrt{\frac{x-3}{-2}} + 1$ and $-\sqrt{\frac{x-3}{-2}} + 1$. Therefore, $f(x)$ is not one-to-one.